Question 1:

Confirmation of Compton effect through experimentation and analysis of X-rays scattered by perspex

Introduction

Following the introduction of the quantum theory of light by Albert Einstein, Compton proposed a theory of X-ray scattering as a quantum phenomenon, now known as Compton effect or Compton scattering. According to the photoelectric effect, electromagnetic radiation liberates electrons when it hits a metal (The Open University, 2021a). However, unlike the classical theory of light, the energy is transferred to electrons from each light quantum independently, resulting in the velocity, and hence the kinetic energy, of the ejected electrons to not be affected by the intensity of the incident light. (Einstein, 1905) This was supported by observations as the emission of electrons was found to be dependent on a threshold frequency of the incident photon regardless of its intensity. (The Open University, 2021b) Through his experimental work, Compton concluded that incident photons are scattered by the electrons in certain directions and that the wavelength of the scattered photons, contrarily to classical electromagnetism, are greater than the wavelength of the incident photons. He also suggested that, as particles, photons carry a directed momentum inversely proportional to the wavelength. (Compton, 1923a) Compton's experimental work was a significant contribution to the modern wave-particle theory of light as opposed to the classical wave theory of light.

By replicating Compton's experiment using an X-ray apparatus and a perspex target to scatter the photons, this investigation aims at confirming the consistency of the current observations and results with Compton scattering formula and hence establishing that the photons carry momentum.

Methods and results

Figure 1 shows that when an X-ray photon with energy E_{in} and momentum $\boldsymbol{p}_{\text{in}}$ hits an electron at rest in the target, the electron gets ejected, carrying energy E_{e} and momentum $\boldsymbol{p}_{\text{e}}$, and scatters the photon at a scattering angle θ. The scattered photon carries energy E_{out} and momentum $\boldsymbol{p}_{\text{out}}$.

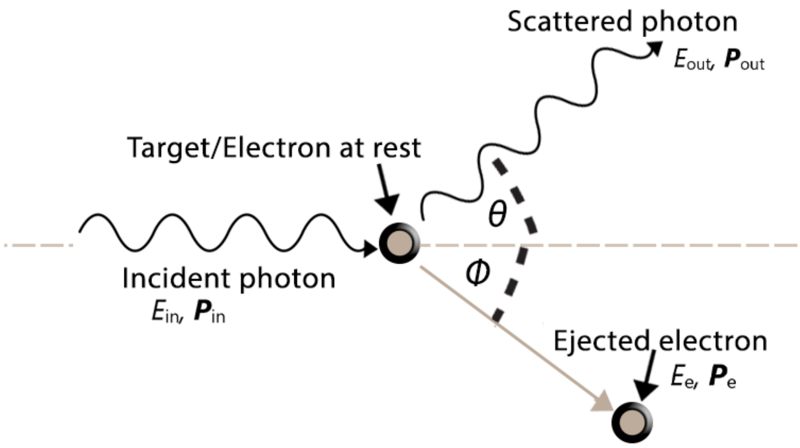

Figure 1. Scattering of a photon after hitting a static electron (Compton scattering)

As photons have zero mass, the momentum cannot be obtained from the classical law $\boldsymbol{p} = mv$. It can be calculated using de Broglie equation:

$\boldsymbol{p} = \frac{h}{\lambda}$, where h is Planck's constant and λ is the wavelength in metres. (The Open University, 2021b)

The momentum is a vector that changes upon the change in direction. Accordingly, the scattering of a photon changes its momentum and hence the wavelength increases with the decrease in the momentum. (Compton, 1923a) As the energy $E = hf$, where f is the frequency and the speed of light $c = \lambda f$, the increase in the wavelength of the photon will be associated with a loss of energy, i.e. the energy of the scattered photon E_{out} will be less than the incident photon energy E_{in}. Thus, Compton's theory can be confirmed by measuring and comparing the energies of the incident photon and scattered photon. The consistency can also be confirmed by comparing the observed measurements to the measurements predicted using Compton formula: $E_{out} = \frac{E_{in}}{1 + \frac{E_{in}}{m_e c^2}[1 - cos\theta]}$, Eq. 1

where $m_e c^2$ is the mass energy of the electron in keV, and θ is the scattering angle. (The Open University, 2021b)

For the generation of X-rays and measurement of their energy, an X-ray apparatus is used. Figure 2 shows the main components of the apparatus. Upon switching on, the X-ray tube starts to generate X-rays. The X-ray beam then gets collimated through the collimator to be incident on the perspex target only. Although Compton used a graphite target in the original experiment, the scattering from perspex is not significantly different. (The Open University, 2021b) As shown, there is a hole in the holder and the target to allow for the passing of X-rays directly to the detector without scattering during the calibration scan. The detector gives electronic pulse with a hight proportional to the photon energy. To measure the energy of the photon, it is connected to a pulse-hight analyser. The detector angle (the scattering angle) θ is adjustable for the purposes of calibration and scattering.

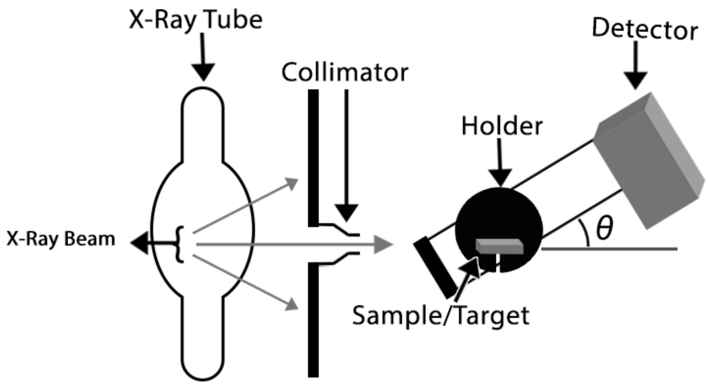

Figure 2. Schematic of the X-ray apparatus used in the experiment

Table 1. Details of the scan made during the experiment session

Scan type	Detector angle θ (˚)	Target angle (˚)	Voltage (kV)	Curren t (mA)	Gate time (s)	Channels
Quick scan	0	-90	30	0.1	20	512
Calibration scan	0	-90	30	0.2	300	512
Scattering scans	150	20	30	1	300	512
	130	20	30	1	300	512
	110	20	30	1	300	512
	90	20	30	1	300	512
	70	20	30	1	300	512
	50	20	30	1	300	512
	35	20	30	1	300	512

The X-ray apparatus was controlled remotely for only 90 minutes. With the time constraint, precision was compromised to allow for scans for longer time and hence reduce the errors. Table 1 shows the details of the scans that were entered in the apparatus remote interface to carry out the scans. First, we took a quick scan to check whether the current should be increased to give sufficient count per second. The count was less than 100/s, so we increased the current to 0.2 mA to take the calibration scan. This was followed by the scattering scans at a current of 1 mA and a range of 7 detector (scattering) angles to allow for comparison and significant results and Compton effect. We collected data for one scattering angle above 130 and one between 30 and 40 to ensure a coverage of a range sufficient for the analysis.

Figures 3-5 show initial graphs of the data collected for the greatest and smallest scattering angles, and an angle in between, compared to the calibration (unscattered) spectrum.

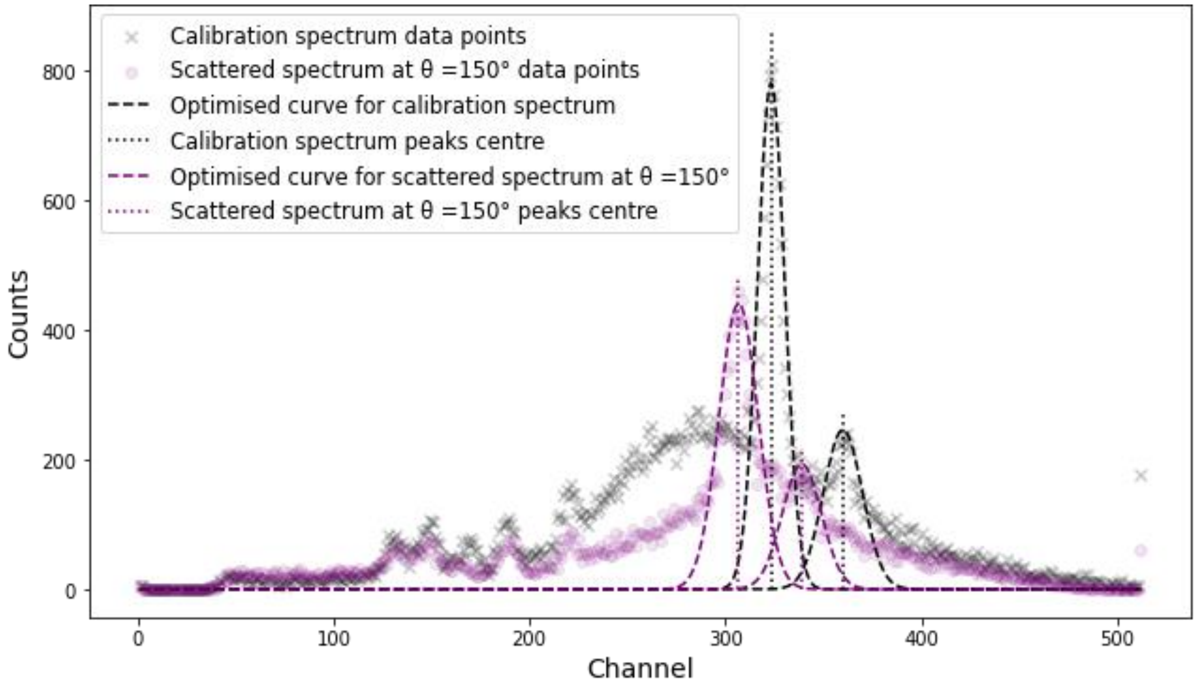

Figure 3. Scattered spectrum at θ =150° compared to calibration (unscattered) spectrum

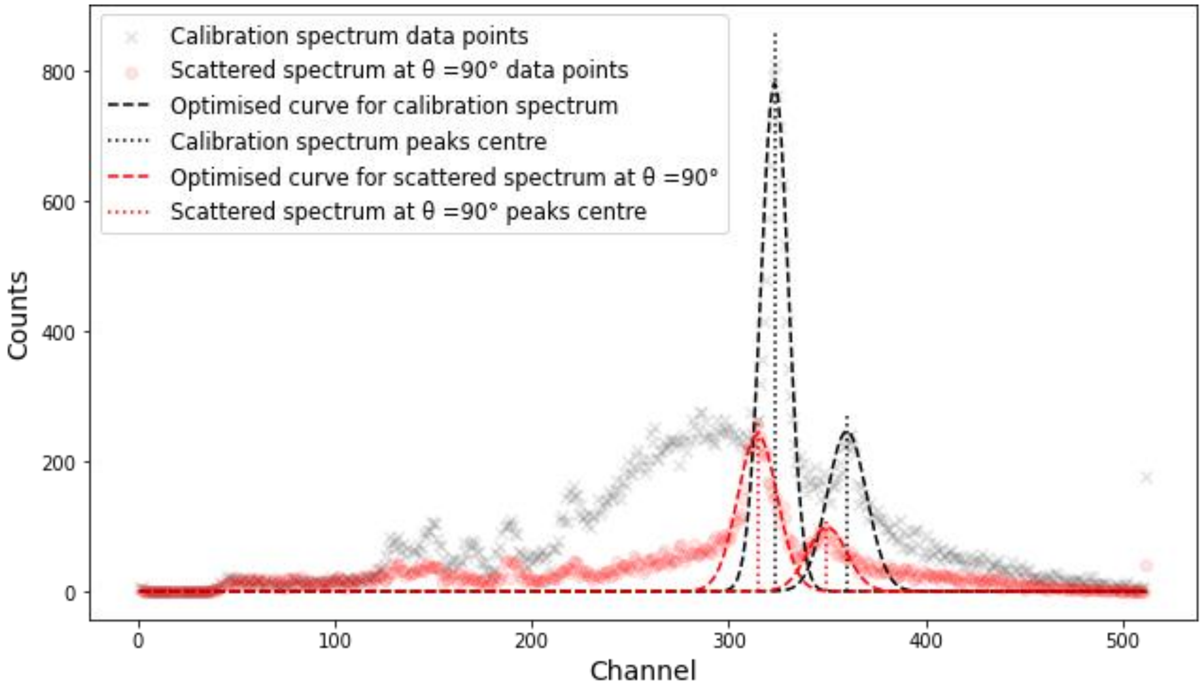

Figure 4. Scattered spectrum at θ =90° compared to calibration (unscattered) spectrum

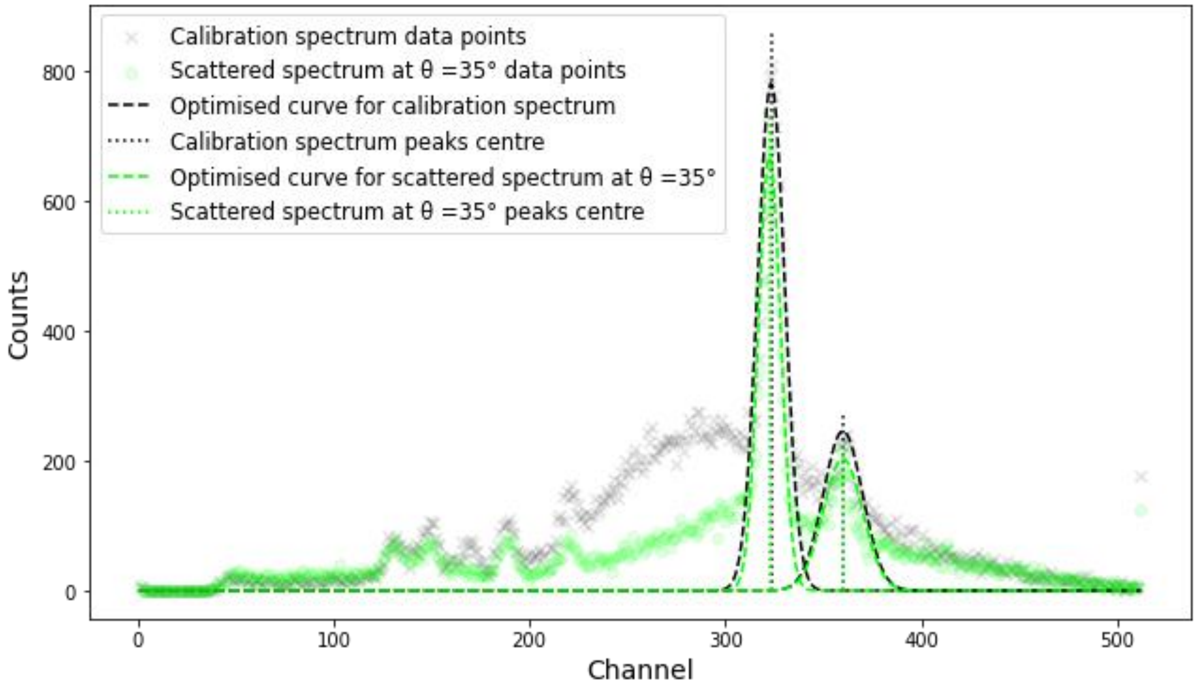

Figure 5. Scattered spectrum at θ =35° compared to calibration (unscattered) spectrum

Figures 3-5 do not show K_α channel error as reported in Tables 2 and 3 for the graphs to be clear and easy to read.

In this experiment, the energy of the scattered photon is indirectly measured. The analyser registers the photons at certain channels, the higher the channel, the higher the photon energy. By determining a relationship between both quantities (the channel and the energy) the energy measurements of the observed photons can be obtained. To know which channel corresponds to which energy, calibration is conducted using unscattered X-rays with known energies, K_α and K_β, which are 17.4 keV and 19.6 keV respectively (The Open University, 2021c). The following equation expresses the relationship between both quantities:

$$E = E_1 + (n-n_1) \times \frac{(E_2 - E_1)}{(n_2 - n_1)}, \qquad \text{Eq. 2}$$

where E is the energy of the observed photon, E_1 is the energy of K_α, E_2 is the energy of K_β, n the channel of the observed photon, n_1 is the energy of K_α, and n_2 is the energy of K_β.

Table 2. Results of the analysis of the calibration (unscattered) scans

$\boldsymbol{\theta}$	0 ˚
K_α peak channel n_1	323
Error in K_α peak channel	0.384
K_β peak channel n_2	360
Error in K_β peak channel	1.98
Energy of K_α E_1	17.4 keV
Energy of K_β E_2	19.6 keV

From the calibration (unscattered) spectrum, it can be visually estimated that the K_α is registered at channel 320 and K_α and K_β at 360 approximately. Table 2 shows more accurate data calculated using a Python programme, which gave estimates for the channels errors as well based on upper and lower values and the base value for the channel.

To compare the results to the values predicted by Compton formula (Eq. 1), processing only the measurements of scattered K_α peak at the different angles was sufficient. Another Python program was used to first calculate the error in the channel of scattered K_α peaks. By substituting in Eq.2 with the values in Table 2 and the values of scattered K_α peaks as well as their associated errors, the energies of the scattered photons in keV can be obtained. Table 3 shows the results of the processing the scattering data.

Table 3. Results of the analysis of the scattering scans

θ (°)	K_α peak channel n	Error in channel	E_{out} (keV)	Error in E_{out} (%)	1-cos θ	Error in 1-cos θ	$1/E_{out}$ (keV^{-1})	Error in $1/E_{out}$ (keV^{-1})	Predicted E_{out} (keV)	1/ Predicted E_{out} (keV^{-1})
150	307	0.437	16.5	2.10	1.87	0.00873	0.0608	0.00125	16.4	0.0611
130	309	0.462	16.6	2.10	1.64	0.0134	0.0604	0.00127	16.5	0.0607
110	311	0.388	16.7	2.09	1.34	0.0164	0.0599	0.00125	16.6	0.0601
90	315	0.418	16.9	2.10	1.00	0.0175	0.0591	0.00124	16.8	0.0594
70	318	0.368	17.1	2.09	0.66	0.0164	0.0585	0.00122	17.0	0.0588
50	321	0.310	17.3	2.08	0.36	0.0134	0.0579	0.00120	17.2	0.0582
35	323	0.223	17.4	2.07	0.18	0.0100	0.0575	0.00119	17.3	0.0578

By substituting with the calibration data and the values K_α peak channel n in Eq. 2, the values of E_{out} were obtained with the associated error in percentage. For the purposes of further analysis, cos θ was calculated for the 7 scattering angles. As the detector angle was entered to the nearest degree, the error in the angle is 1 degree. To allow for a comparison between the observations and Compton predictions, 1-cos θ and the corresponding error were calculated for each angle to be substituted with in Compton formula (Eq. 1) to obtain the predictions. The energy value of unscattered K_α was substituted with for E_{in} in Compton formula, with the mass energy of the electron being 511 keV. Following the rules of error combination, all calculations were done using a Python programme, and the values of Predicted E_{out} were obtained accordingly.

To make the comparison easier and Compton effect clearer, the predicted and observed values of $1/E_{out}$ were calculated as shown in Table 3 and plotted with the absolute value of the error in the observed E_{out} in Figure 6.

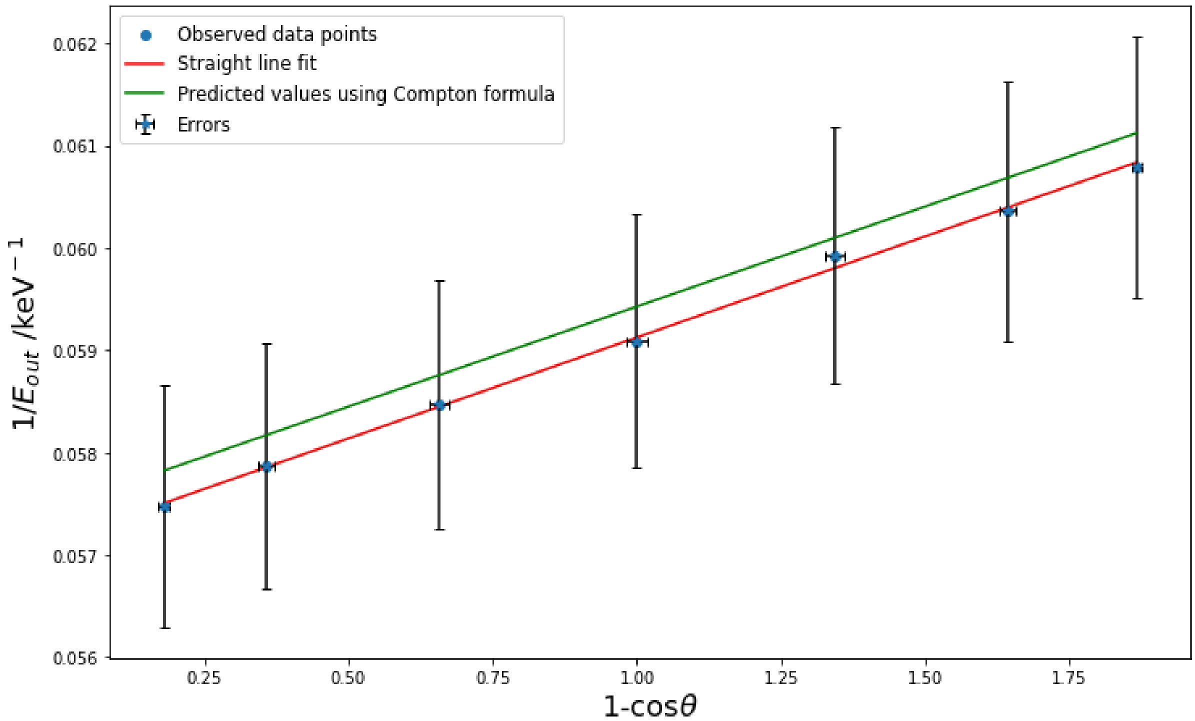

Figure 6. Observed and predicted $1/E_{out}$ against 1-cosθ

The graph shows the line of the energies predicted using Compton formula passing through the error bars of the observed energies.

The above results are reported to 3 sig. fig. They were obtained by processing one scan for each angle. The precision of the results could have been improved by doing multiple scans and processing the average. However, this was not possible due to the limited time. The count was also below 100/s for most of the scattering scans. The current was set at maximum current of the X-ray apparatus, 1 mA (The Open University 2021c), and we could not increase the voltage beyond 30 kV to avoid heating.

Discussion and interpretation

It can be concluded visually from Figures 3-5 that the interaction with the perspex target resulted in a scattering and a loss of energy increasing with the increase in the detector (scattering) angle θ. As the loss of energy corresponds to an increase in the wavelength, this is consistent with Compton's conclusion that the wavelength increases when the scattering angle θ increases. (Compton, 1923b) The peaks of the data collected at $\theta = 35^{\circ}$ almost matched the unscattered data peaks. This could be attributed to the low loss of energy at small scattering angles and the peak being registered by the analyser at the same channel as the unscattered K_{α}. Table 3 confirms the visual observations as the K_{α} photon experienced the largest loss of energy (17.4 – 16.5 = 0.9 keV) when it was scattered at $\theta = 150^{\circ}$.

The centrelines in Figures 3-4 also show that photons with higher energies K_{β} (the shorter peaks at higher channels) experience a greater loss of energy as the distance between the centrelines of the scattered and unscattered K_{β} peaks is a bit larger compared to the distance between the centrelines of the scattered and unscattered K_{α} peaks.

The consistency of the observed results with the values predicted by Compton formula (Eq.1), as shown in Table 3 and Figure 6, further confirms Compton's theory. Like the observed values, the predicted values showed a decrease in the energy of the scattered photon. Compton suggested that each of the X-ray quanta is scattered by one electron. When the scattering electron bounces upon the change in the X-ray momentum, the energy of the X-ray decreases with an increase in the wavelength (Compton, 1923b) based on the expression of momentum as $\boldsymbol{p} = \frac{hf}{c}$ (Compton, 1923a), which can be expressed in de Broglie equation $\boldsymbol{p} = \frac{h}{\lambda}$, as $c = f\lambda$.

Compton's work, supported by the current investigation, proves the wave-particle duality of light and that some phenomena cannot be explained with the classical wave theory of light.

Conclusions

- Compton's experimental work was replicated using an X-ray apparatus and a scattering target made of perspex instead of graphite.
- The energies of the scattered photons were indirectly measured and were obtained through calibration using X-rays with known values (K_{α} = 17.4 keV and K_{β} = 19.6 keV).
- The results showed a loss of energy and hence an increase in the wavelength, which contradicts with classical electromagnetism prediction.
- The loss of energy is proportional to the energy of the incident photon.
- The results were consistent to within the reported errors with the values of E_{out} predicted by Compton formula.
- The experiment supports Compton's theory of X-ray scattering as a quantum phenomenon, with photons having momentum $\boldsymbol{p} = \frac{h}{\lambda}$.

References and acknowledgments

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